1d plume model based on equations in Joe Kordzi's Basic program. The solution is not coupled, but instead individual components are added together. Individual parts include regional groundwater velocity, operational plume, buoyancy, dispersion, and diffusion.

Facility: Corsicana Technologies
Case: 1 - k=191 md 100years

1. Define Units

$cp := .01 \cdot poise$

$g = 9.807\,m \cdot s^{-2}$ $\quad md := 7.32441 \cdot 10^{-8} \cdot \dfrac{\dfrac{ft^3 \cdot cp}{sec}}{ft^2 \cdot \dfrac{psi}{ft}}$ $\quad gal := 0.1336894 \cdot ft^3$ $\quad acre := 43560 \cdot ft^2$

2. Reservoir and 10,000 Year Plume Demonstration Parameters

$\rho go := 61.97 \cdot \dfrac{lb}{ft^3}$ $\quad \rho gi := 61.61 \cdot \dfrac{lb}{ft^3}$

Light plume

$k := 191 \cdot md$ permeability $\quad \rho gi = 0.987 \dfrac{gm}{cm^3}$ injectate density

$\phi := 0.207$ porosity $\quad \rho go = 0.993 \dfrac{gm}{cm^3}$ formation fluid density

$h := 36 \cdot ft$ net thickness

$cumvolume := 50 \cdot \dfrac{gal}{min} \cdot 30 \cdot yr$ $\quad dip := 200 \cdot \dfrac{ft}{2.355 \cdot mi}$ taken from structure map

$\mu := 0.479 \cdot cp$ viscosity $\quad cumvolume = 7.889 \times 10^8\, gal$ cumulative injection volume

$\Delta t := 100 \cdot yr$ plume drift time $\quad \theta := 0.92146586\, deg$ $\quad \sin(\theta) = 0.016082$

$\alpha T := 16 \cdot ft$ transverse dispersivity $\quad D0 := 4.8 \cdot 10^{-5} \cdot \dfrac{cm^2}{s}$ free water diffusivity

$\alpha L := 160 \cdot ft$ longitudinal dispersity $\quad CRF := 1 \cdot 10^{-3}$ concentration reduction factor

$Vdrift := 0 \cdot \dfrac{ft}{yr}$ regional groundwater velocity $\quad \tau := 1$ tortuosity

3. Operational Plume Radius and Area

$$operational\_plume\_radius := \sqrt{\frac{cumvolume}{\pi \cdot \phi \cdot h}}$$

$operational\_plume\_radius = 2.123 \times 10^{3}\, ft$ calculated operational plume radius

$Area\_of\_plume := \pi \cdot operational\_plume\_radius^{2}$

$Area\_of\_plume = 14153336.4\, ft^{2}$ calculated operational plume area

$Area\_of\_plume = 324.916\, acre$

4. Movement due to Regional Ground Water Velocity

$\Delta t = 100\, yr$ $\quad Vdrift := 0 \cdot \frac{ft}{yr}$

$Ground\_water\_movement\_distance = Vdrift \cdot \Delta t$

$Ground\_water\_movement\_distance = 0\, ft$ calculated movement from regional velocity

5. Movement due to density drift from bouyancy

$$Den1 := 4 \cdot \pi \cdot \sqrt{\alpha T \cdot \alpha L} \cdot k \cdot \left|\rho gi - \rho go\right| \cdot g \cdot \sin(\theta) \cdot \Delta t$$

$Den1 = 0.344\, kg \cdot s^{-1}\, ft$

$Den2 := \phi^{2} \cdot \mu \cdot Area\_of\_plume$

$Den2 = 88.542\, kg \cdot s^{-1}\, ft$

$$Den3 := 4 \cdot \pi \cdot \frac{\sqrt{\alpha T \cdot \alpha L}}{Area\_of\_plume \cdot \phi}$$

$Den3 = 2.17 \times 10^{-4}\, ft^{-1}$

$$Density\_drift\_distance := \frac{\left[\left[1 + \left(\frac{Den1}{Den2}\right)\right]^{0.5}\right] - 1}{Den3}$$

$Density\_drift\_distance = 8.9\, ft$ calculated plume movement from bouyant drift

6. Movement due to dispersion and diffusion

$\tau := 1$ tortuosity $\quad D0 = 4.8 \times 10^{-9}\, m^{2} \cdot s^{-1}$ free water diffusivity

$CRF = 1 \times 10^{-3}$ concentration reduction factor

$L := Density\_drift\_distance + operational\_plume\_radiu$

$L = 2131.47ft$ operational plume radius and buoyant plume length

$standard\_deviation := [2 \cdot [(\alpha L \cdot L) + (D0 \cdot \tau \cdot \Delta t)]]^{0.5}$

$standard\_deviation = 826.1ft$

Iterative Calculations for error function

$i := 0 .. 6000$ $\quad X_i := 0.01 + (i \cdot 0.001)$ $\quad X_0 = 0.01$

$Error\_CRF := 2 \cdot (CRF)$ $\quad Error\_CRF = 0.002$

$Error\_function_i := erfc(X_i)$ $\quad Error\_function_0 = 0.98871658444415$

$Difference_i := Error\_function_i - Error\_CRF$ $\quad Difference_0 = 0.987$

$$Converge(X, tol) := \left| \begin{array}{l} j \leftarrow 0 \\ \text{while } \ Difference_j > tol \\ \quad j \leftarrow j + 1 \\ X_j \end{array} \right.$$

Programming loop to determine argument - solve inverse complementary error function

Define arrays for loop

$Converge(X, 0.0000000000001) = 2.186$

$Z := Converge(X, 0.0000000000000)$

$Z = 2.186$ The argument of the inverse complementary error function

$\Delta r := \sqrt{2} \cdot Z \cdot standard\_deviatior$ from the following equation: $CRF := erfc\left(\frac{\Delta r}{standard\_deviation \cdot \sqrt{2}}\right)$

$\Delta r = 2553.8ft$ diffusive and dispersive plume component

7. Summary of Results

$operational\_plume\_radius = 2122.5ft$ calculated operational plume radius

$Ground\_water\_movement\_distance = 0ft$ calculated movement from regional velocity

$Density\_drift\_distance = 8.9ft$ calculated plume movement from bouyant drift

$\Delta r = 2553.8ft$ diffusive and dispersive plume movement total_plume_distance := operational_plume_radius+ Ground_water_movement_distance+ Density_drift_distance

total_plume_distance = 4685.3ft   total plume movement from all effects

↑ distance from well the .001 dispersion factor
is expected to be after 100 years